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We examine the behaviour of a closed oscillating universe filled with a homogeneous scalar field and find that, contrary to naive expectations, such a universe expands to larger volumes during successive expansion epochs. This intriguing behaviour introduces an arrow of time in a system which is time-reversible. The increase in the maximum size of the universe is closely related to the work done on/by the scalar field during one complete oscillatory cycle which, in turn, is related to the asymmetry in the scalar field equation of state during expansion and collapse. Our analysis shows that scalar fields with polynomial potentials $V(\phi) = \lambda\phi^q$, $q > 1$ lead to a growing oscillation amplitude for the universe: the increase in amplitude between successive oscillations is more significant for smaller values of q. Such behaviour allows for the effective recycling of the universe. A recycled universe can be quite old and can resolve the flatness problem. These results have strong bearing on cosmological models in which the role of dark matter is played by a scalar field. They are also relevant for chaotic inflationary models of the early universe since they demonstrate that, even if the universe fails to inflate the first time around, it will eventually do so during future oscillatory cycles. Thus, the space of initial conditions favourable for chaotic inflation increases significantly.

I. INTRODUCTION

The idea of a cyclical oscillating universe—one that is continuously reborn from the ashes of a previous existence—finds expression in the philosophical and cultural beliefs of many ancient civilizations [1–4].

Within the framework of modern relativistic cosmology, oscillating models (analytically continued through the big bang singularity) arise naturally as exact solutions of the Einstein field equations for a spatially closed universe consisting of a perfect fluid. Since all expansion-contraction cycles in such models are identical, one might feel that an oscillating universe containing an infinite number of cycles would be infinitely old, somewhat resembling, on the average, a steady-state model. Dissipative processes leading to entropy growth, however, change this picture radically. As originally demonstrated by Tolman [5], the growth of entropy increases the total volume of the universe at the maximum of each expansion cycle; this observation has several important consequences, some of which are summarised below.

(i) Tolman strongly felt that the possibility of thermodynamically recycling the universe would have a "liberalizing action on our general thermodynamic thinking" since it would dispel the notion that "the principles of thermodynamics necessarily require a universe which was created at a finite time in the past and which is fated for stagnation and death in the future" [5]. Thus, the oscillating universe was seen to present a credible alternative to the idea of the thermodynamic heat death postulated by nineteenth century physicists and popular in this century as well. (The latter may still be possible in a flat/open universe.)

(ii) As demonstrated by Zeldovich and Novikov [6], the increase in entropy from cycle to cycle suggests that an oscillating universe could not have had an infinitely long total duration since, given the present (finite) value of its total entropy and postulating that the entropy increase from cycle to cycle is finite, one is led to conclude that the number of cycles preceeding the present one is also finite. An oscillating universe cannot therefore be infinitely old and must have been created, perhaps quantum mechanically, at some point in the past. Since the total mass (energy) of a closed universe is zero, its creation from the vacuum does not violate any known laws of conservation and is therefore possible, in principle [7].

(iii) An important consequence of an oscillating universe with an increasing expansion maximum at every cycle, is that the horizon and flatness problems are gradually ameliorated as the universe grows older, larger, and flatter during each successive expansion cycle. The oscillating universe may thus present a credible alternative to the inflationary scenario in this respect.

An oscillating universe can also have other important cosmological implications. For instance, relics of an earlier expansion epoch may be measurable today [8]. Starobinsky, in his seminal paper on graviton production in an inflationary universe [9], showed that gravity waves would also be created in an oscillatory universe; further, he derived an expression for their amplitude and spectrum which allows us to infer details of previous expansion epochs from measurements made today.

Finally, the idea of an oscillating universe also appears in the quasi-steady-state cosmology of Hoyle, Burbidge, and Narlikar [10,11], in which alternate cycles of expansion and contraction modulate an exponentially expanding background geometry, with the creation of matter being most intense at the minimum of each cycle.

A key issue, which remains at present unresolved, relates to physical mechanisms which can cause the universe to bounce at the end of each cycle. The singularity theorems of Penrose and Hawking suggest that a singular state necessarily arises at the end of contraction in a closed Friedman–Robertson–Walker (FRW) universe described by general relativity, if matter satisfies certain 'energy conditions' [12]. However, it is not clear whether, at very high curvatures $\mathcal{R} \sim l_{\rm P}^{-2}$, matter would behave as it does under 'normal' conditions when the space-time curvature is much smaller than the Planck value, $\mathcal{R} \ll l_{\rm P}^{-2}$. Furthermore, at $\mathcal{R} \sim l_{\rm P}^{-2}$, particle production and vacuum polarisation effects are likely to be significant; the resulting vacuum expectation value of the energy-momentum tensor $\langle T_{ik} \rangle$ need not satisfy the above energy conditions, due to which the space-time metric could very well 'bounce' without ever reaching a singular state [6,13]. Finally, we should mention that a 'bouncing' universe may also arise in quantum cosmology or as a consequence of the duality conditions which are generic features of superstring-inspired cosmological models. The limiting curvature hypothesis of Markov [14] (see also [15]) might also lead to a time symmetric bounce. Other physical mechanisms leading to singularity avoidance are discussed in [16,17].

The bouncing of the universe in the vicinity of the singularity, $a = 0$, is perceived less dramatically by extending the range of the scale factor to negative values. Since only the absolute value of the scale factor, $|a|$, has direct physical meaning, such an extension is quite legitimate (see, for instance, [18] and Fig. 1). The notion of a 'bounce' is then replaced by the more natural notion of a continuous 'passage' of the scale factor through the value of $a = 0$ to the region of negative values $a < 0$. This also can be viewed as the change of the (unphysical?) spatial orientation of the universe after its passage through zero value of the scale factor [19]. We will use this notion in our formulation of 'natural' conditions at the bounce. Of course, the dynamics of the universe in the close vicinity of the point $a = 0$ is beyond classical description, even in this model.

This paper presents a radical departure from most previous work on oscillating models in which entropy production associated with dissipative processes led to the growth of the expansion maximum of each cycle. Our results show that an alternate mechanism which does not require entropy production exists, also leading to increasing oscillatory cycles. We demonstrate that the presence of a massive scalar field (in a closed FRW universe), under certain reasonable conditions at the bounce, gives rise to growing expansion cycles, the increase in expansion amplitude being related to the work done by/on the scalar field during the expansion/contraction of the universe. (The presence of other matter fields, in addition to the scalar field, does not affect our conclusions, as long as interactions between such fields and the scalar are sufficiently weak.) Our results have important consequences for the inflationary universe scenario, one of the standard paradigms of cosmology.

II. SCALAR FIELDS IN A CLOSED UNIVERSE.

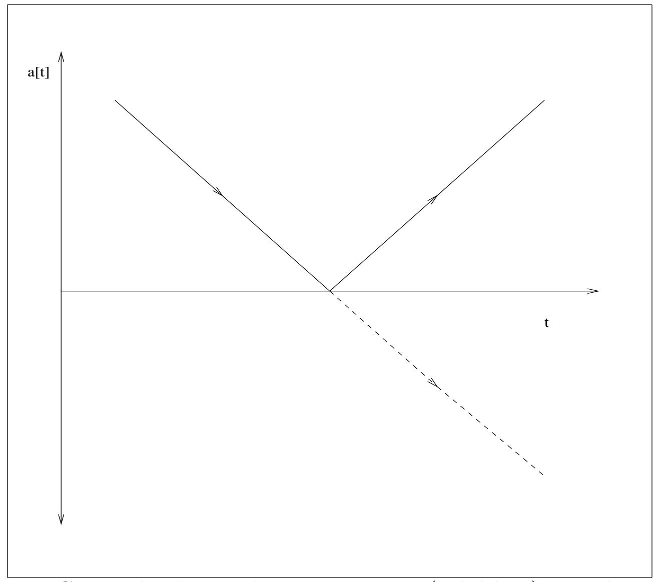

FIG. 1. The 'bounce' at $a = 0$, $t = 0$ (solid line) is replaced by a continuous transition of the scale factor through $a = 0$ towards negative values of a (dotted line). Both representations are physically equivalent since only the *absolute* value of a has physical significance.

The recent detection of anisotropy in the cosmic microwave background on degree scales appears to favour a closed FRW universe with $\Omega_{\rm total} = 1.11 \pm 0.07$ [20]. A spatially closed space-time is described by the metric

$$ds^2 = dt^2 - a^2(t) \left(\frac{dr^2}{1 - r^2} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2 \right) , \quad (1)$$

where $a(t)$ is the cosmic expansion factor; units in which the speed of light $c = 1$ are used throughout this paper. In a closed universe, the Einstein equations acquire the well-known form

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{1}{a^2} , \quad (2)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) , \quad (3)$$

where ρ and P are, respectively, the total energy density and pressure of the various components of the universe. For a radiation-dominated universe, we have $P = \rho/3$, and the field equations can be solved exactly in terms of the conformal time coordinate $\eta = \int dt/a(t)$, to yield the solution

$$a(\eta) = A \sin\eta , \qquad t = A(1 - \cos\eta) , \quad (4)$$

which describes a semicircle in the a–t plane. The solution for a matter dominated universe (with $P = 0$) is

$$a(\eta) = A(1 - \cos\eta)\,, \qquad t = A(\eta - \sin\eta)\,, \tag{5}$$

which describes a cycloid. After solution (4) is extended periodically in time, both (4) and (5) describe a periodic evolution with an infinite number of identical expansion–contraction cycles, as shown in Fig. 2.

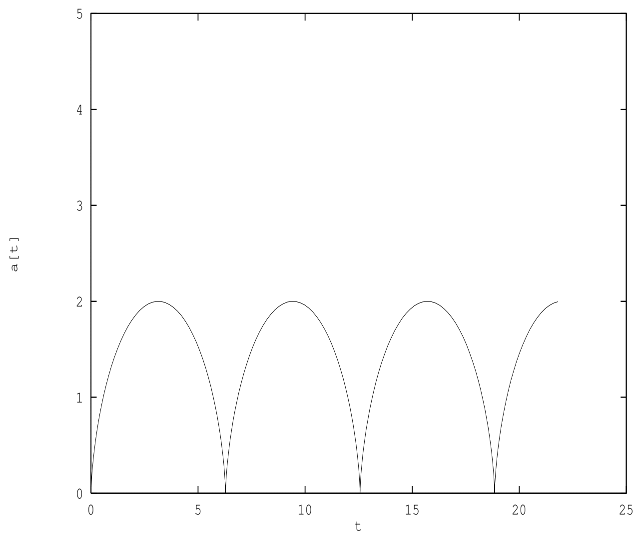

FIG. 2. The cycloid solution for a matter dominated universe. Note that the amplitude of the cycloid does not increase from one cycle to the next.

The presence of a bulk viscosity ζ results in a modification of the cosmological fluid pressure P to

$$P = P_0 - 3\zeta H\,. \tag{6}$$

Here, P_0 is the equilibrium pressure and $H = \dot{a}/a$ is the Hubble parameter. (An example of bulk viscosity is provided by a fluid in which energy is easily exchanged between translational and rotational/internal degrees of freedom, an example being a gas of rough spheres [21].) From Eq. (6) one finds that, during expansion, $H > 0$ and $P < P_0$, whereas during collapse, $H < 0$ and $P > P_0$. This asymmetry during the expanding and contracting phases results in the growth of both energy and entropy, in the words of Tolman [5], "if the pressure tends to be greater during a compression than during a previous expansion, as would be expected with a lag behind equilibrium conditions, an element of fluid can return to its original volume with increased energy and hence also with increased entropy." The increase in energy makes the amplitude of successive expansion cycles larger enabling the universe to spend "a greater and greater proportion of its period in a condition of lower density . . . even though a return to higher densities would always occur."

Although Tolman linked the asymmetry in pressure during expansion and collapse to the production of entropy, we will show that such an asymmetry also arises for non-dissipative Lagrangians such as those describing a massive scalar field in an FRW space-time. In this case, the asymmetry in pressure leads to a significant increase in the energy of the scalar field and results in an increase of the maximum volume of the oscillating universe.

The Lagrangian density for a scalar field has the form

$$\mathcal{L} = \frac{1}{2} g^{ij}\partial_i\phi\partial_j\phi - V(\phi)\,. \tag{7}$$

The 0-0 Einstein equation for a homogeneous scalar field in the closed FRW universe (1) becomes

$$H^2 = \frac{8\pi G}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right] - \frac{1}{a^2}\,, \tag{8}$$

and the energy density and pressure of the scalar field are, respectively,

$$\rho_\phi \simeq \frac{1}{2}\dot{\phi}^2 + V(\phi)\,, \qquad P_\phi \simeq \frac{1}{2}\dot{\phi}^2 - V(\phi)\,. \tag{9}$$

The scalar-field equation of motion is

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV}{d\phi} = 0\,. \tag{10}$$

The motion of ϕ arises in response to the dual action of the accelerating force $dV/d\phi$ and the damping term $3H\dot{\phi}$. In the chaotic inflationary scenario with $V \propto \phi^n$ ($n = 2, 4$), for sufficiently large values of ϕ, namely, for $\phi \gtrsim m_{\rm P}$, where $m_{\rm P}$ is the Planck mass, the damping caused by the expansion of the universe ($H > 0$) settles the scalar field into a 'slow-roll' regime during which $\frac{1}{2}\dot{\phi}^2 \ll V$ and $P \simeq -\rho$. Since the equation of state of the scalar field mimics that of the cosmological constant, the expansion of the universe is inflationary, $a \propto \exp\int H(t)dt$. Exactly the reverse situation arises when the universe contracts. In this case, we have $H < 0$, and the term $3H\dot{\phi}$ now accelerates the motion of ϕ instead of damping it, as it did during expansion. As a result, the kinetic energy of the scalar field becomes much larger than its potential energy, $\frac{1}{2}\dot{\phi}^2 \gg V$, and the resulting equation of state becomes $P = \rho$ (sometimes called the equation of state of 'stiff' matter). Consequently, the scalar field in a closed universe satisfies two generic regimes [22]

$$P \simeq -\rho \quad \text{for} \quad H > 0 \quad (\text{expansion})\,, \tag{11}$$

$$P \simeq \rho \quad \text{for} \quad H < 0 \quad (\text{contraction})\,. \tag{12}$$

For polynomial potentials, $V \propto \phi^n$, these two regimes are separated by an epoch during which the scalar field oscillates about its minimum value while its equation of state mimics that of dust: $\langle P\rangle = 0$ ($n = 2$), or radiation: $\langle P\rangle = \langle\rho\rangle/3$ ($n = 4$). (The time average is taken over many oscillations of the field.) It is interesting that the quantity P/ρ, when plotted as a function of the expansion factor, resembles a hysteresis curve. The area enclosed by the curve is related to the work done by/on the scalar field during the expansion of the universe $\delta W = \oint PdV$.

If one postulates that the universe 'bounces' during contraction, then one can expect the 'work done' δW during a given expansion cycle to be converted into 'expansion energy', resulting in the growth in amplitude of each successive expansion cycle. An estimate of the increase in the expansion maximum can be obtained from the following elementary considerations: setting $H = 0$ in (2) we obtain

$$\rho_* = \frac{3}{8\pi G a_{\rm max}^2}\,, \tag{13}$$

where ρ_* is the density of the universe at the expansion maximum. Since the total mass of a closed universe is $M = 2\pi^2 a^3 \rho$, this gives

$$M = \frac{3\pi}{4G} a_{\rm max}\,. \tag{14}$$

Equating the increase in energy, $\delta E = \delta M$, to the work done during a single expansion-contraction cycle, $\delta E = \delta W$, we obtain

$$\Delta a_{\rm max} = \frac{4G}{3\pi} \oint P dV\,, \tag{15}$$

which relates the increase in the expansion maximum between two successive cycles $\Delta a_{\rm max}$ to the work done $\delta W = \oint P dV$. (The increase in energy δE is brought about at the expense of the gravitational field energy, since the total energy of a closed universe is identically zero [5,23,6].)

We probe further implications of this analysis by studying equations (8)–(10) numerically for the class of chaotic potentials $V(\phi) = \lambda\phi^q$, $q > 0$, and $V(\phi) = V_0 \exp(-\mu\phi)$. The potential $V(\phi) \propto \phi^q$ has been discussed both in the context of inflation [24] and as a candidate for cold dark matter [25]. Exponential-based classes of potentials are known to have important cosmological consequences both within the inflationary framework [26] and as candidates for quintessence and cold dark matter [27]. (An early use of the exponential potential in the context of a closed universe may be found in [28].) Before describing our results, it is necessary to first discuss the type of 'bouncing' conditions which are imposed at the start of each expansion-contraction cycle. As remarked earlier, it is likely that a unified theory of the weak, strong, and gravitational interactions will provide a key to understanding physical processes which operate during the strong-curvature regime of a contraction cycle. In the absence of such a theory, we will do the next best thing and set conditions which we feel are both simple and natural and may therefore arise in a future 'physically complete' theory of the early universe.

The conditions imposed at the bounce are, $a \to a$, $\dot a \to -\dot a$, $\phi \to \phi$, and $\dot\phi \to \dot\phi$, (the bounce is assumed to occur instantaneously). Thus, the effect of the bounce is to reverse the direction of motion of the universe (since $\dot a \to -\dot a$), with all other quantities remaining unchanged. In the introduction, we mentioned the possibility of extending the range of the scale factor to negative values and then replacing the idea of a 'bounce' by the somewhat more appealing idea of 'passage' to negative values of the scale factor. This viewpoint leads to the following reasonable conditions at the bounce: $a \to -a$, $\dot a \to \dot a$, $\phi \to \phi$, $\dot\phi \to \dot\phi$, which is equivalent to the 'conventional' prescription $a \to a$, $\dot a \to -\dot a$, $\phi \to \phi$, $\dot\phi \to \dot\phi$ adopted in this paper. Further, we assume that the bounce arises at scales at which quantum gravitational effects become important, i.e., at Planck scales. Two possibilities then exist for the *location* of the bounce:

(i) The bounce occurs when the *curvature* of the universe becomes comparable to the Planck scale, in other words, the bounce takes place at a fixed value $a_{\rm min} \simeq l_{\rm P}$ of the scale factor.

(ii) The bounce occurs when the *energy density* in the scalar field crosses the Planck energy, i.e., when $\frac{1}{2}\dot\phi^2 + V(\phi) \simeq m_{\rm P}^4$.

Both the above possibilities are considered in the current work, with results shown in Figs. 3 and 4, respectively, for the boundary conditions (i) and (ii), and for the potential $V(\phi) = \frac{1}{2}m^2\phi^2$. In Fig. 3, we follow the expansion factor through one expansion-contraction cycle; it can be seen that the universe inflates at the beginning of the second cycle, resulting in a very large universe and a correspondingly large value of $a_{\rm max}$.

Next, in implementing condition (ii), we will work with what we feel is the 'worst case scenario' for inflation, namely, we set

$$\frac{1}{2}\,\dot\phi^2\Big|_{a=a_{\rm min}} \simeq m_{\rm P}^4 \gg V(\phi)|_{a=a_{\rm min}}\,, \tag{16}$$

$$\phi|_{a=a_{\rm min}} < \phi_* \simeq m_{\rm P}\sqrt{\Omega/6\pi} \tag{17}$$

at the start of the first expansion cycle. Note that $V(\phi) = \frac{1}{2}m^2\phi^2$ and $\Omega = 8\pi G\rho_\phi/3H^2 > 1$.

The initial conditions (16) and (17) guarantee that the condition for accelerated expansion $\dot\phi^2 < V(\phi)$ is not met and thus ensure that the inflationary regime *does not commence*. Our results, shown in Fig. 4, are interesting; we find that, although the universe is prevented from inflating during the first cycle, its amplitude during subsequent cycles increases, resulting finally in inflation! The physical reasons for a growing oscillation amplitude are simple to understand: the large kinetic energy of the scalar field during the collapse phase of a cycle drives ϕ to regions higher up on the potential at the commencement of the next expansion cycle, until the field finally reaches an amplitude $\phi > \phi_*$, which is large enough to make the universe inflate.

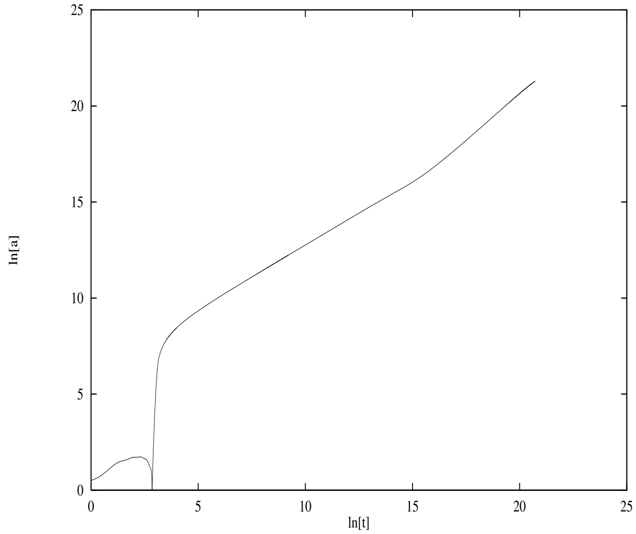

FIG. 3. The expansion factor for an oscillating universe satisfying the boundary condition (i).

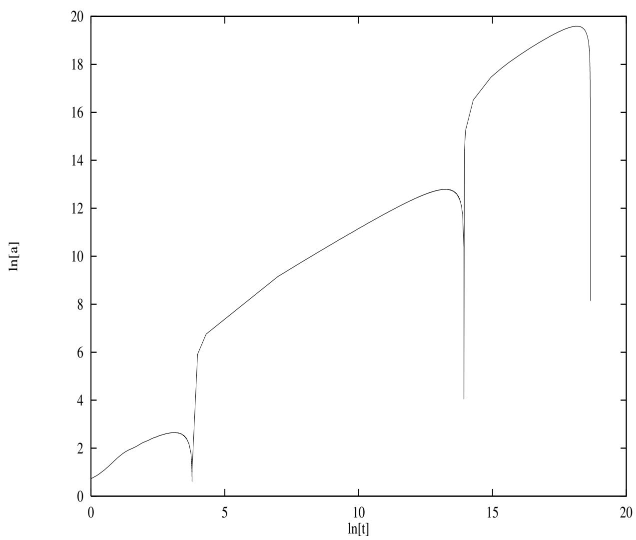

FIG. 4. The expansion factor for an oscillating universe satisfying the boundary condition (ii). Note the monotonic increase in successive expansion maxima (plotted on the logarithmic scale).

The hysteresis curves corresponding to the first oscillation of the expansion factor in Figs. 3 and 4 are shown in Figs. 5 and 6, respectively. Suprisingly, we find that the phenomenon of 'hysteresis' is present even when the conditions (11) and (12) are no longer met. The resulting increase in oscillation amplitude emphasises the fact that, although the universe may not inflate during its 'first attempt', it will eventually do so during subsequent cycles! This result considerably enhances the scope and appeal of inflationary models by showing that inflation could begin from a much broader set of initial data, provided the universe bounces when it reaches a high density. (It should be pointed out that, for scalar fields originating from larger initial values $\phi|_{a=a_{\min}} > \phi_*$, there will be greater hysteresis in the equation of state and a larger associated growth in the volume of the universe at maximum expansion.)

We thus find that a growth in the maximum expansion amplitude can be achieved without any recourse to an entropy generating mechanism. In fact, the field equations (8)–(10) are non-dissipative and time-reversible; moreover, the bouncing conditions imposed are also time-reversible, in the sense that the time-reversed evolution will respect the same bouncing conditions. One therefore arrives at the following important conclusion: *time-asymmetry in the evolution of a closed universe can be achieved even with time-reversible field equations and bouncing conditions!* This result may appear counter-intuitive, especially when compared with the behaviour of the periodic solutions (4) and (5), which are exactly time-symmetric with respect to the expansion maximum of each cycle. The reason behind the growing amplitude of successive expansion cycles has to do with the fact that convex potentials (which have so far been examined) have a well defined minimum about which the scalar field oscillates. Rapid oscillations of ϕ with frequency $m \equiv d^2V/d\phi^2 \gg H$ cause the field to 'mix' in its phase-space $\{\phi, \dot{\phi}\}$ so that, at the time of recollapse, its location in $\{\phi, \dot{\phi}\}$ is almost completely uncorrelated with its location in $\{\phi, \dot{\phi}\}$ before the onset of oscillations. As a result, the chance that the scalar field will roll up its potential during recollapse along exactly the same trajectory down which it rolled during expansion is essentially zero. This accounts for the fact that the equation of state during contraction is $P = \rho$ and not $P = -\rho$, which it would have been for exactly time-symmetric expansion–collapse. The time-asymmetric behaviour of ϕ is clearly shown in Figs. 5 and 6, in which the solid line showing the equation of state P/ρ during expansion is displaced with respect to the broken line showing P/ρ during collapse. For exact time reversal, no such difference between expansion and collapse would have been present; the solid and broken lines would overlap, and the 'hysteresis' seen in the figures would not occur. The global time-asymmetry of the evolution of an oscillatory universe in our model is also a direct consequence of the time-asymmetric physical conditions at the bounce, in which the sign of $\dot{a}$ is reversed while the sign of $\dot{\phi}$ is not. It should be noted at this point that, had we adopted the bouncing condition $\dot{\phi} \to -\dot{\phi}$ for the scalar-field velocity, then every two subsequent cycles would obviously be time-symmetric with respect to the corresponding bouncing point, and the cosmological evolution as a whole would thus be periodic with period comprising two subsequent cycles.

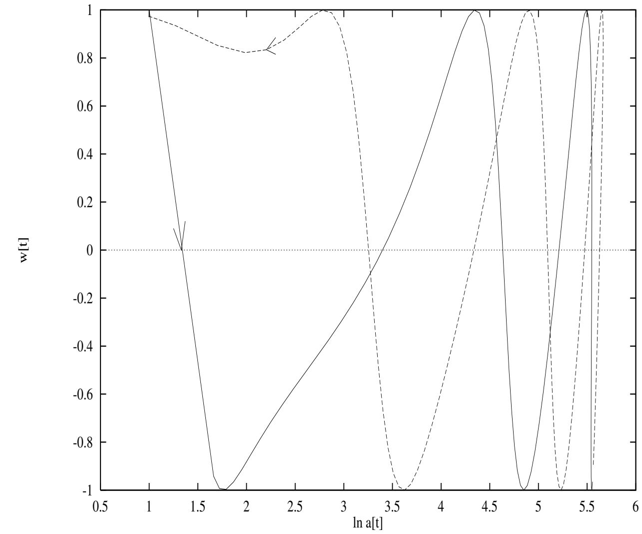

FIG. 5. The scalar-field equation of state $\omega = P/\rho$ is shown for the first cycle of Fig. 3. Solid/broken lines correspond to the expanding/collapsing epochs. Note that during the pre-oscillation and post-oscillation epochs, the behaviour of P/ρ resembles a hysteresis curve, which contributes significantly to the total 'work done' $\delta W = \oint P dV$.

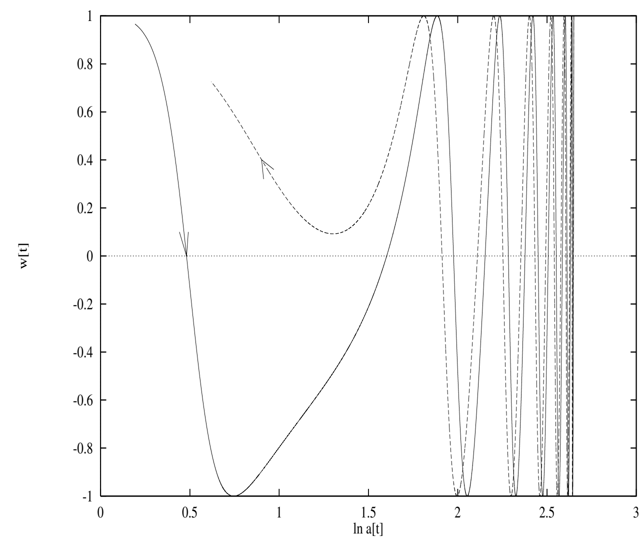

FIG. 6. The hysteresis curve corresponding to the first cycle of Fig. 4.

The above argument is supported by our analysis of exponential potentials $V = V_0 \exp(-\mu\phi)$ for which we find no hysteresis in the equation of state P/ρ and no increase in the amplitude of the expansion factor through successive oscillations. The example of an exponential potential is easy to analyse because the evolution of the scalar field proceeds with time towards large positive values of ϕ and, at such values of the scalar field, the potential approaches an identical zero—a trivial case for which the evolution is periodic and hysteresis is obviously absent. Interestingly enough, hysteresis reappears when we modify the potential to $V = V_0 [\cosh(\lambda\phi) - 1]$, which has been proposed as a candidate for cold dark matter in [29]. This potential has the following limiting forms: $V \propto \exp(-\lambda\phi)$ for $|\lambda\phi| \gg 1$, $\phi < 0$; and $V \propto \lambda^2\phi^2$ for $|\lambda\phi| \ll 1$. In this case, although the amplitude of successive cycles increases, the increase is much smaller than for a purely $\lambda^2\phi^2$ potential. This observation supports our conclusion that hysteresis, and the accompanying increase in the amplitude of successive expansion maxima, depends crucially upon the ability of the scalar field to oscillate. The absence of a well-defined minimum in the exponential potential prevents the field from oscillating, its behaviour is therefore monotonic and, in the absence of 'mixing', the scalar field equation of state P/ρ does not show any hysteresis. For potentials characteristic of chaotic inflation, $V \propto \phi^q$, $q > 0$, the extent of 'hysteresis' and the accompanying increase in successive expansion maxima depend sensitively upon the exponent q—reflecting the steepness of the 'chaotic' potential ϕ^q. For larger q, the amount of hysteresis is smaller, as is the increase in the value of successive maxima of the expansion factor.

It is interesting to follow the value of the cosmological density parameter Ω as the universe oscillates. From the definition

$$\Omega - 1 = (aH)^{-2}, \tag{18}$$

we find that a larger value of the expansion factor a (at identical H) will result in a value of Ω which is closer to unity (during successive cycles). This is borne out by the results of our numerical analysis shown in Fig. 7, in which the value of Ω measured at identical values of $t - t_{\rm min} \sim H^{-1}$ is shown for successive expansion-contraction cycles [the boundary condition is (ii); the corresponding expansion factor is shown in Fig. 4]. We find that Ω approaches unity more closely and for a longer duration during successive cycles thereby gradually ameliorating the flatness problem. (During the third expansion cycle, the universe inflates and Ω approaches unity to great accuracy.)

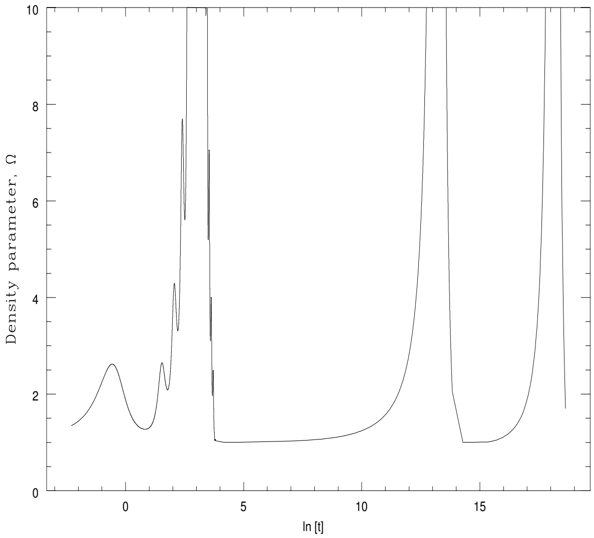

FIG. 7. The density parameter Ω as a function of time ($\Omega \to \infty$ as the universe recollapses). Note that the time axis is given in logarithmic units; the oscillations seen (in Ω) in the first cycle are thus not visible in the later cycles due to the compression of the scale.

The time derivative of equation (18) yields

$$\dot{\Omega} = H\Omega(1-\Omega)[1+3\omega(t)]\,, \tag{19}$$

where $\omega(t)$ is the general time-dependent equation of state $\omega = P(t)/\rho(t)$. Equation (19) shows that the behaviour $\Omega = \text{const}$ arises if either (i) $\Omega = 1$ or (ii) $\omega(t) \equiv -1/3$. The value $\Omega = 1$ is the well-known critical point of the Friedmann equations. The fact that the equation of state $\omega \equiv -1/3$ also results in a universe evolving with an *unchanging value of* Ω is not so well known, even though this result is completely general and independent of any assumptions about the curvature of the universe. It is interesting that oscillations of the scalar field [with $\omega(t)$ varying between $\omega = +1$ and $\omega = -1$] induce oscillations in $\Omega(t)$, since every time $\omega(t)$ passes through the critical value $\omega_c = -1/3$, the value of $\dot{\Omega}$ changes sign. Thus, close to the time of recollapse, a large increase in the value of Ω is modulated by small oscillations, as shown in Fig. 7. The growth in Ω is due to the increase in the importance of the curvature term relative to the scalar field density in (2), whereas small modulations in Ω are due to changes in the equation of state of the oscillating scalar field.

From (18) we find the following simple expression for the total volume of a closed universe

$$V = \frac{2\pi^2}{(\Omega_{\rm total}-1)^{3/2}}\, V_H\,, \tag{20}$$

where $V_H = H^{-3}$ is the current Hubble volume. Observations of anisotropies in the cosmic microwave background on degree scales appear to favour a closed universe with $\Omega_{\rm total} = 1.11 \pm 0.07$ [20]. By substituting this into (20), we get $258 < V/V_H < 2467$. The total volume of our universe could therefore be several thousand times larger than its causal horizon! We also obtain an estimate of the value of the expansion factor at recollapse ($a_{\max}$) and the corresponding recollapse 'redshift' ($z_{\rm recoll}$) by setting to zero the (future) value of the Hubble parameter [27]

$$H(z) = H_0(1+z)\left[1 - \Omega_{\rm total} + \sum_\alpha \Omega_\alpha (1+z)^{\gamma_\alpha}\right]^{1/2}, \tag{21}$$

where Ω_α is the fraction corresponding to matter of type α in $\Omega_{\rm total}$, $\Omega_{\rm total} = \sum_\alpha \Omega_\alpha$, $\gamma_\alpha = 1 + 3w_\alpha$, and $1+z = a_0/a$. Specialising to a CDM dominated universe and assuming for simpicity that *all* the matter in the universe is pressureless so that $\Omega_{\rm total} = \Omega_m$, we get

$$H(z) = H_0(1+z)\left[1+\Omega_m z\right]^{1/2} = 0\,, \tag{22}$$

which leads to $z_{\rm recoll} = -1/\Omega_m$ or

$$\frac{a_{\max}}{a_0} = \frac{\Omega_m}{\Omega_m - 1}\,. \tag{23}$$

Substituting $\Omega_m = 1.11 \pm 0.07$, we obtain $6.5 \lesssim a_{\max}/a_0 \lesssim 26$, i.e., the size of the universe at recollapse is ~ 10 times larger than its present size. The corresponding age of the universe at recollapse is

$$t_{\rm recoll} = \int_{-1/\Omega_m}^{\infty} \frac{dz}{H(z)(1+z)}\,, \tag{24}$$

giving

$$t_{\rm recoll} = \frac{\pi}{2H_0}\frac{\sqrt{\Omega_m}}{[\Omega_m - 1]}\sqrt{-\left(\frac{\Omega_m}{1-\Omega_m}\right)} \tag{25}$$

i.e. $t_{\rm recoll} \sim 400 - 4000$ billion years (for $h = 0.5$).

The late-time behaviour of a $\Lambda_{\rm CDM}$ model suggested by recent supernova observations [30] is more complex since, if Λ is a constant, the universe need not recollapse at all but could continue expanding forever [31,27]. Quintessence fields which generate a time-dependent Λ-term result either in an ever-expanding universe (if w_Q always remains $\lesssim -1/3$) or in recollapse (if the current acceleration of the universe is a transient phenomenon).

III. DISCUSSION AND CONCLUSIONS.

We have analysed the behaviour of a massive scalar field in a closed FRW universe and shown that oscillations of the field about the minimum of its potential can lead to an asymmetry in its equation of state $\omega = P_\phi/\rho_\phi$ during the expansion and collapse epochs. This asymmetry is reflected by the presence of a hysteresis-like feature in ω which is, in turn, related to the amount of 'work done' by/on the scalar field during a given expansion-contraction cycle. An important consequence of this effect is that the imposition of bouncing conditions at an appropriate early time causes the work done during a given expansion cycle to be converted into expansion energy, resulting in the growth in amplitude of each successive expansion maximum in an oscillating universe. The increase in the value of the expansion maximum results in a recycled universe which is considerably longer-lived and more flat during each successive expansion epoch. Thus, the flatness problem is gradually ameliorated in this model.

Our results also have a direct bearing on the issue of initial conditions in cosmology. We have shown that even in the worst-case scenario in which the curvature term dominates, causing the universe to collapse prematurely without inflating, subsequent cycles will ensure that the value of the scalar field (or 'level of the potential') increases at the commencement of each expansion cycle, until the value of ϕ and $V(\phi)$ become large enough for inflation to occur. Thus, even if the universe did not inflate the first time around, it will eventually do so, due to the growing amplitude of ϕ and $V(\phi)$ at the commencement of each new expansion cycle. Inflation in closed models therefore turns out to be remarkably robust, provided the universe bounces when it reaches a high density [32].

We note, finally, that observations of intermediate-angle anisotropies in the Cosmic Microwave Background made by the BOOMERanG satellite appear to favour a closed universe, with $\Omega \simeq 1.11 \pm 0.07$ indicated by a combined analysis of BOOMERANG-98 and MAXIMA-1 data [33,20]. Furthermore, although we have not specified the exact nature or origin for our scalar field, one might be tempted to view it either as the inflaton or an inflationary relic. Indeed, the possibility that relic scalar fields might play the role of dark matter in the universe is very tempting and has been discussed in [34,25,29].

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